Mathematical Methods in Scientific Models -82833

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Course outline (2 credit points)

The aim of the course is to familiarize graduate students in scientific disciplines such as Oceanography, Meteorology, Hydrology, Geology, Biology, etc. with mathematical methods that are frequently employed in modeling phenomena in these fields. The course is a 2 credit point (14 weeks, 2 hours per week) and it includes homework assignments, in-class work on problems and a final independent research project. The grade will be determined based on homework assignments and class-work (20%) and the final research project (80%) in which each student will apply the methods to a specific model (selected from the students' area of interest/expertise).

A previous mathematical knowledge in differential equations is not required or assumed but the students must be willing to spend the time and learn the basic methods of solutions o these equations, including their applications, as part of the homework assignments. The course has no textbook and is based on my own notes (so class attendance is a must for those wishing to get credit for it). Many of the examples will involve simple algebraic equations but others will be ordinary differential equations that describe time-dependent processes (again - the required background will be covered in class).

The topics covered in the course are:

1. Small is NOT necessarily negligible: The neglect of terms preceded by a small number is not a trivial matter even though we do it routinely in attempting to simplify the (fairly complex) equations that govern most interesting processes in Science. I will show counter examples, where the neglect of such terms has dramatic effects on the solution of a given (algebraic or differential). We will also attempt to understand when and why this happens and what we should look out for in neglecting terms preceded by a small parameter. Some basic perturbation methods will be described and applied.
2. Transformation to dimensionless variables: This method reduces the number of free parameters that appear in the equation and helps us clarify to what extent a parameter is indeed small. I will show that although the process of nondimensionalizing the equation by scaling the (dependent and independent) variables is not unique there are some choices of the scaling that can be very helpful in applying approximate, perturbation schemes to the nondimensional equation while other choices of scales are not conducive to perturbation procedure.
3. Regular perturbation methods: Power series expansion (regular Taylor expansion) and singular perturbation series (Frobenius series expansion)
4. Conservation laws and how to use them: In cases when it is possible to obtain a conserved quantity from a set of time-dependent equations the analysis of the solution can be greatly simplified by reducing the number of equations in the set (at the expense of adding a free model parameter whose value is set by the initial conditions)
5. Eigenvalue problems: Many cases exist in which a given problem has meaningful solutions only for some values of the model parameters while for other values the solutions are void of information. We will apply simple numerical methods (since the analysis is, in general, too complex) to identify such values and decipher the structure of the (physically relevant) solutions at these values.
6. WKB method – the mathematical rational, its application to initial or boundary value problems. We'll see examples in which this approximate method yields the exact solution of fairly complex differential problems